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PRESENTACIÓN MURAL

Thermodynamics of regular black hole interiors

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Abstract. A regular black hole is represented by a singularity-free solution of the Einstein's field equations. One possible set of regular black hole solutions has the geometry of the space-time described by Schwarschild's solution at large radii and by a de Sitter-like solution at small radii. Solutions of this kind can be found for some choices of the equation of state in a static, spherically symmetric configuration. Adopting the equation of state suggested by Mbonye and Kanzanas (2005), the model of the interior of the black hole consists of matter fields with sound speed bounded by the speed of light. The matter transits smoothly between normal matter and a core of a "quintessence-like" fluid with an equation of state that approaches $p = -\rho$ when $r \to 0$. In this work we address the question of the thermodynamical behavior of the matter that constitutes the interior of this non-singular black hole model. We derive the general equations of the thermodynamic quantities for an arbitrary density profile and adjust the results to the specific regular black hole. Then, we discuss a possible physical interpretation of the state of regular black hole interiors.

Resumen. Un agujero negro regular se representa por una solución de las ecuaciones de Einstein libre de singularidades. La geometría del espacio-tiempo para un posible conjunto de estas soluciones es la solución de Schwarzchild para radios grandes, mientras que cerca del origen la geometría del espacio-tiempo es de de Sitter. Dependiendo de la elección de la ecuación de estado, este tipo de soluciones pueden tener una configuración estática y esféricamente simétrica. La ecuación de estado que utilizamos en este trabajo fue propuesta por Mbonye y Kazanas (2005): el modelo del interior del agujero negro regular consiste en materia cuya velocidad del sonido es siempre menor que la velocidad de la luz. El estado de la materia cambia en forma continua y suave desde materia normal a materia extraña cerca del origen con una ecuación de estado del tipo $p = -\rho$ para $r \to 0$. En este trabajo estudiamos el comportamiento termodinámico de la materia en el interior del agujero negro no singular a partir de las ecuaciones generales de la termodinámica. Por último, discutimos una posible interpretación física del estado del agujero negro regular.

1. Introduction

A special type of regular black hole solution is characterized by the presence of matter at the center of a static configuration with an equation of state of the form $p = -\rho$. The repulsive behavior of the matter prevents complete gravitational collapse, as in the case of the so-called 'gravastars'. See Camenzind (2007), Sect. 8.7, for a detailed discussion of the motivation for using this particular equation of state and density profile. The resulting description of the space-time geometry is singularity-free.The main goal of the present work is to study the thermodynamic properties of the interior black hole region of such a space-time and to provide a possible physical interpretation.

2. Regular black hole

2.1. General features

A static spherically symmetric geometry can be described by the most general line element:

$$ds^2 = -B(r)dt^2 + A(r)dr^2 + r^2(d\theta^2 + \sin^2(\theta)d\phi^2). \tag{1}$$

In order to obtain the line element that describes a regular black hole, it is necessary to calculate the expressions for $B(r)$ and $A(r)$. We adopt the equation of state suggested by Mbonye and Kazanas [1]:

$$p_{\mathsf{r}} = \left[\alpha - (\alpha+1)\left(\frac{\rho}{\rho_{\mathsf{max}}}\right)^2\right]\left(\frac{\rho}{\rho_{\mathsf{max}}}\right)\rho, \tag{2}$$

with $\alpha = 2.2135$. The equation was constrained to obtain a model of the interior of the black hole consisting of matter fields with sound speed bounded by the speed of light. The physical state changes smoothly between normal matter and a core of a "exotic matter" fluid with an equation of state that approaches $p = -\rho$ when $r \to 0$.

The solution to the Einstein field equations $G^{\mu}_{\nu} = -8\pi T^{\mu}_{\nu}$, for the metric and the equation of state (2) takes the form [2]:

$$ds^2 = -B(r)dt^2 + \left(1 - \frac{2m(r)}{r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{3}$$

$$B(r) = \exp\left\{-\int_{r_0}^{r} \frac{2}{r'^2}\left[m(r') + 4\pi r'^3 p_{\mathsf{r}'}\right]\left[\frac{1}{\left(1 - \frac{2m(r')}{r'}\right)}\right] dr'\right\}, \tag{4}$$

where the mass $m(r)$ is given by $m(r) = 4\pi\int_0^r \rho(r')r'^2 dr'$.

Equation (3) describes the geometry of the spacetime of a regular black hole with matter and a de Sitter core. We can derive from the g_{rr} component of the metric that this model presents an event horizon at R=$2M$. Here, R is the radius at which the total mass M of the black hole is contained.

In order to study the thermodynamic properties of this space-time we work with the normalization of the density profile suggested by Dymnikova [3]:

$$\rho(r) = \rho_{\mathsf{max}} e^{-8\frac{r^3}{R^3}}. \tag{5}$$

Here, $R^3 = 8r_g r_0^2$, r_g is a length scale of order of the Schwarzschild radius and $r_0 = (3/8\pi\rho_{\mathsf{max}})^{1/2}$.

2.2. Thermodynamics

The temperature of the matter as a function of the radius can be calculated from the first law of thermodynamics:

$$TdS = d(\rho V) + pdV. \tag{6}$$

From the Equation (6) we derive the following expression:

$$dp = \frac{\rho + p}{T} dT. \tag{7}$$

Replacing Equations (2) and (5) into (7) we obtain:

$$\frac{T}{T_{\mathsf{sup}}} = \left[1 + \alpha e^{-8r^3/R^3} - (\alpha+1)e^{-24r^3/R^3}\right]^{4/3} e^{\Xi(r)}, \tag{8}$$

$$\Xi(r) = \frac{2}{3}\int_0^{e^{-8r^3/R^3}} \frac{\alpha d(\rho/\rho_{\mathsf{max}})}{1 + \alpha(\rho/\rho_{\mathsf{max}}) - (\alpha+1)(\rho/\rho_{\mathsf{max}})^3}. \tag{9}$$

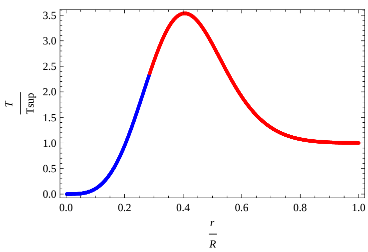

Figure 1. Temperature as a function of radial coordinate.

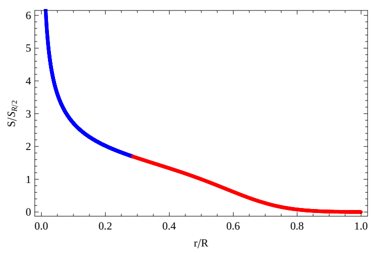

Figure 2. Entropy density as a function of radial coordinate.

Substituing (7) in (6), we obtain an expression for the entropy up to an additive constant. We then estimate the entropy density for the matter inside the black hole obtaining:

$$\frac{s}{s_{\mathsf{R}/2}} = \frac{(\rho/\rho_{\mathsf{max}})[1 + \alpha(\rho/\rho_{\mathsf{max}}) - (\alpha+1)(\rho/\rho_{\mathsf{max}})^3]^{-1/3}}{0.2076 e^{(2/3)\int_0^\rho \alpha d\rho[\rho_{\mathsf{max}} + \alpha\rho - (\alpha+1)(\rho^3/\rho_{\mathsf{max}}^2)]^{-1}}}. \tag{10}$$

In order to calculate the speed of sound as a function of the energy density, we solve $(v/c)^2 = dp/d\rho$:

$$\left(\frac{v}{c}\right)^2 = 2e^{-8r^3/R^3}\left[\alpha - 2(\alpha+1)e^{-16r^3/R^3}\right]. \tag{11}$$

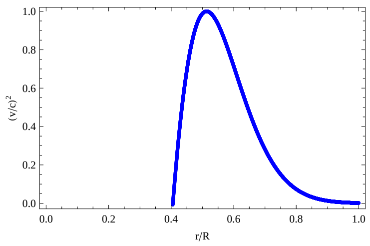

Figure 3. Velocity of sound as a function of radial coordinate.

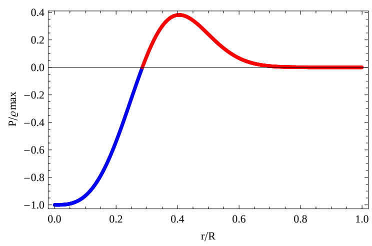

Figure 4. Pressure as a function of radial coordinate.

The result is depicted in Figure 3. Substituing (5) in (2), we obtain an expression for the pressure as a function of the radial coordinate. The result is shown in Figure 4.

3. Discussion

The results of the previous section show that new interesting features arise in this model.

In Fig. 1 we observe that the temperature tends to absolute zero close to the core. Furthermore, from Fig. 2 we can see that the entropy density diverges at the origin. This behaviour can be understood as the impossibility of the system to access to different macrostates. In other words, there is no space of microstates compatible with the macrostate of the system. Hence, standard entropy cannot be define at r=0, since statistical mechanics breaks down there.

From Fig. 3, we see that the sound speed is zero for the same value of r at which the pressure is maximun. In addition, the speed of sound takes complex values in the region $r < 0.4R$. The latter can be derived from the negative slope of the state function shown in Fig. 4. The sound waves cannot propagate in the region $r < 0.4R$ because a variation in pressure causes an expansion rather than a compression of the fluid. Because of the repulsive behaviour of the matter, this region is opaque to sound waves.

We also find that the specific heat is not defined for the same value of r where the sound speed equals zero and the density entropy has a local maximun. This suggests a possible region of instability for the normal matter field. In a forthcoming work we will address the study of the stability in the region where the propagation of sound waves is possible.

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